APPENDIX K

BALLISTIC TRAJECTORY ANALYSIS METHODS

Ballistic trajectory analysis can be applied to selected wreckage pieces to assist in the determination of the breakup sequence. The ballistic trajectory of a wreckage piece can be calculated based on its mass and aerodynamic characteristics, or the Ballistic Coefficient. The Ballistic Coefficient is a function of an object's weight, aerodynamic drag coefficient, and its effective cross sectional area. It should be noted that it is difficult to estimate the attitude of the wreckage pieces during descent. Also, the attitude of the object, relative to the air stream, affects the object's effective cross-sectional area. It is assumed for this analysis that the Ballistic Coefficient for an object is constant. Thus, the ballistic analysis can only be used as reference information to support the flight MH17's break-up sequence analysis.

Dynamic model of the ballistic trajectory

Given an object with mass (*M*) and velocity (*V*). Its flight path is in the *XZ*-plane, making an angle (*Y*) with the direction on the x-axis.

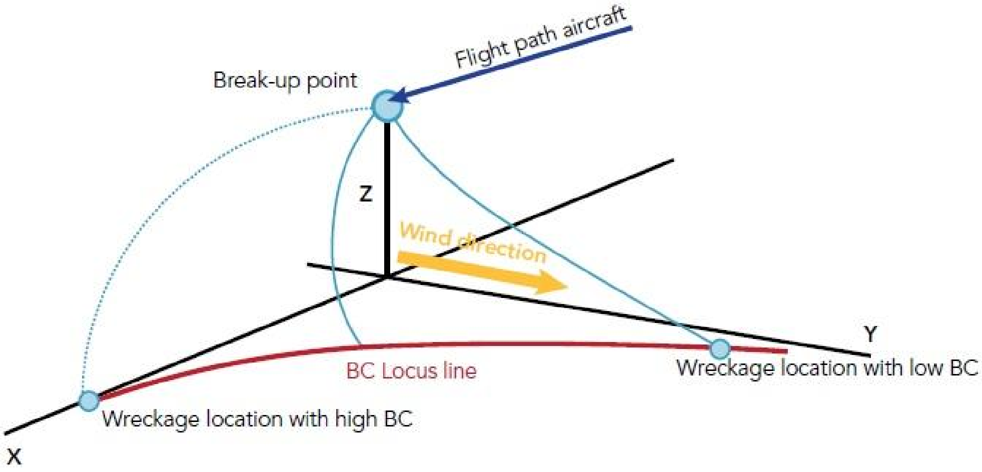


Figure 14: Schematic overview of the effect on the flight path and final position of an object for a high and a low value of the Ballistic Coefficient (BC). The wind is coming along the y-axis in this example. (Source: Dutch Safety Board)

Applying Newton's law, $F = M*a$, the accelerations in the directions of the axes, *X*, *Y* and *Z* can be written as:

Equation 1

$$\dot{V}_x = \frac{-Dg}{W}\cos\gamma\sin\psi + a_x = -\frac{\rho V^2}{2BC}g\ \cos\gamma\sin\psi + a_x$$
$$\dot{V}_y = \frac{-Dg}{W}\cos\gamma\cos\psi + a_y = -\frac{\rho V^2}{2BC}g\ \cos\gamma\cos\psi + a_y$$
$$\dot{V}_z = \frac{Dg}{W}\sin\gamma - g + a_z = -\frac{\rho V^2}{2BC}g\ \sin\gamma - g + a_z$$

Where:

$$BC = \frac{W}{CD * S}; D = {}^{1}/_{2}\rho V^2 S;\ \gamma = \tan^{-1}\left(-\frac{V_z}{V_{xy}}\right); V_{xy} = \sqrt{V_x^2 + V_y^2}$$

$\rho = 0{,}002378e^{-z/30000}, y < 30000\ feet;\ \rho = 0{,}0034e^{-z/22000}, y \geq 30000\ feet$ (Imperial units)

And for the components of the velocity (V) in the directions of the axes:

Equation 2

$$\dot{X} = \int \dot{V}_x dt + V_w \cos\psi_w\ (h)$$
$$\dot{Y} = \int \dot{V}_y dt + V_w \sin\psi_w\ (h)$$
$$\dot{Z} = \int \dot{V}_z dt$$

Where:
γ: flight path angle in the XZ- plane
Ψ : flight path angle in the XY- plane
$\Psi_w(h)$: Angle between x-axis and wind velocity, function of the height above sea level
ρ : air density
a_x, a_y and a_z: longitudinal, lateral and vertical un-modelled accelerations along the three axes *X*, *Y* and *Z*, respectively. These un-modelled accelerations are assumed to be zero for this study
CD: zero-lift drag coefficient
D : aerodynamic drag of the object
M : mass of object
S : reference area of a ballistic object
V: velocity of the object
V_x, V_y and V_z: components of the velocity along the axes *X*, *Y* and *Z*, respectively
VW: wind velocity, function of the height above sea level
W: weight of the object (*M*g*)

It should be noted that in equation 1 the acceleration equals zero along the Z-axis when the terminal velocity is reached. The terminal velocity is defined as the velocity at which aerodynamic drag equals the weight of the ballistic object.

Method 1; trajectory analysis selected wreckage piece
The first method is to calculate the wreckage piece trajectory with a time step simulation from its initial conditions to the ground. The initial condition is described with six parameters: positions (East, North, and altitude), airspeed, flight path angle and heading. After integrating equation 1 in time with the wreckage Ballistic Coefficient and inputting the wind profile, the three axes position variables in equation 2 can be obtained. Applying the initial position and integrating equation 2, the ballistic trajectory of the wreckage piece can be obtained.

For a ballistic trajectory simulation the last recorded altitude, airspeed, and heading parameter values by the Flight Data Recorder are used as the known initial conditions of the simulation. A computer program then outputs a three-dimensional trajectory of the specific wreckage object when it hits the ground. This position is then compared to the wreckage position where it was found.

There are several sources of error in the ballistic trajectory analysis that should be taken into account when interpreting the results. These error sources are not limited to uncertainties in the estimation of:

- the wreckage mass;
- aerodynamic drag coefficient, and
- the wind profile.

The ballistic trajectory analysis assumes that the wreckage pieces fell with a constant Ballistic Coefficient from the moment of separation from the aircraft main body. In fact, wreckage orientation during descent is very difficult to predict. During initial separation, dynamic forces on the wreckage would result in an initial separation condition from a pure ballistic trajectory for a period, which could induce an error in the final descent point. Furthermore, the ballistic trajectory generated does not consider the possible sub-separations of the wreckage pieces. Ballistic trajectory analysis also assumes that wreckage objects separated from the main fuselage at an initial airspeed and with a heading equal to the last recorded flight condition. The accuracy of wind profiles would also impact the accuracy of the results. The wind profile would affect the initial positions of the wreckage items, and may also affect their sequence of separation during the rapid descent.

It is also possible to inverse method 1 and use the wreckage position as the initial condition, hereby calculating the altitude of break-up. In this calculation the errors mentioned previously will also affect this calculation.

Method 2: Ballistic Coefficient locus line
Another way of applying the ballistic simulation is to calculate the ground positions for multiple Ballistic Coefficients thereby creating a locus line. A locus is a shape created by the set of points whose position satisfies a given set of rules. The locus line represents the projected positions of wreckage pieces after break-up given an initial position.

The trajectory of an object with a high Ballistic Coefficient will asymptotically approach its initial heading when the break-up occurres. The trajectory of an object with a low

Ballistic Coefficient will asymptotically follow the wind drift. Thus, for pieces with higher Ballistic Coefficient, the trajectory matching to the recovery location will be more accurate as lighter (low Ballistic Coefficient pieces are influenced more by the wind).

When running this simulation it has the advantage that it creates a representative (locus) line including wind errors but without estimation errors for specific wreckage pieces characteristics (mass, surface area etc). In essence this simulation creates a baseline of expected position after break-up given the initial conditions.

Ballistic Coefficient calculation
During the investigation a video showing falling debris from flight MH17 was published on the internet by unknown persons. By research it was determined that this debris was in fact textile rolls transported as cargo aboard flight MH17. A number of these (partly and fully unrolled) textile rolls were recovered en transported to the Netherlands. Based on the textile retrieved, the full length wound on one roll was estimated at 100 meters. Analysing the video footage a probable location where the video was taken was established. From this location and the known heading of the aircraft five textile rolls were found and identified on satellite imagery in wreckage site 4.

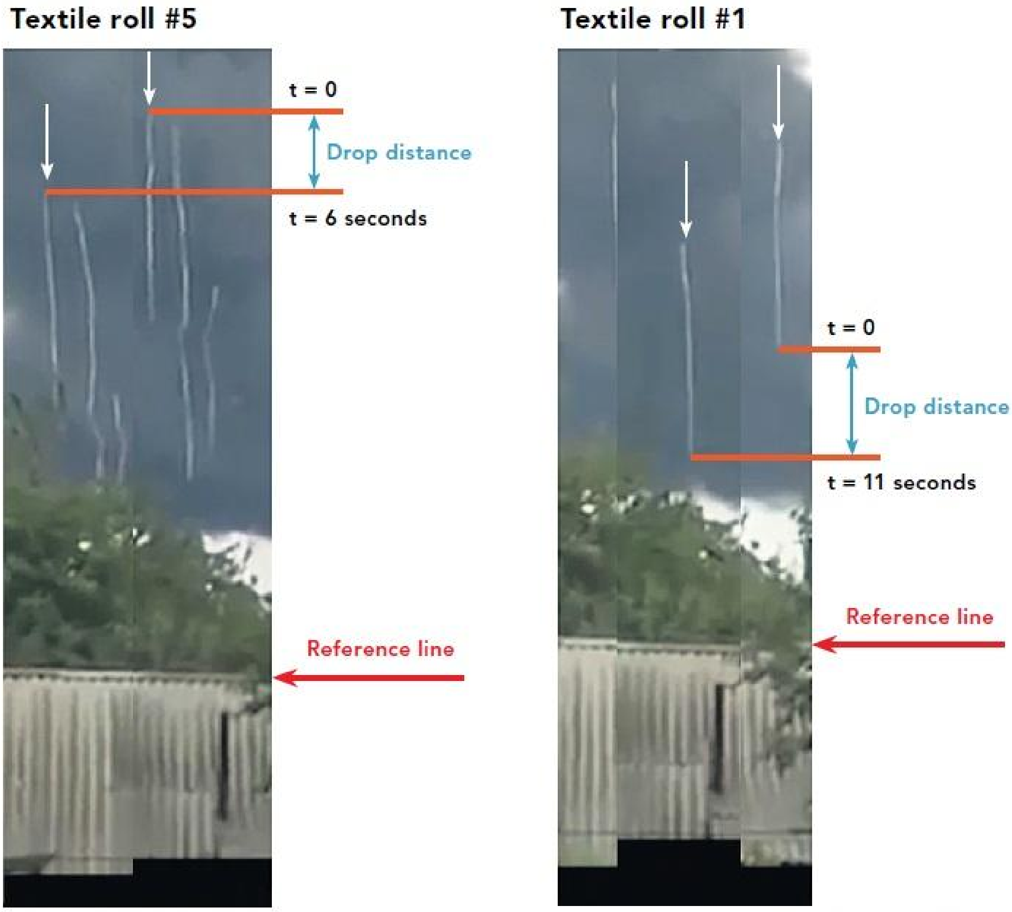
Figure 15: Video showing falling debris (5 white textile rolls) from MH17, the black smoke in the background is from site 6. Image transmitted by various media organisations. (Source: unknown)

The video was further analysed to determine if the Ballistic Coefficient of these textile rolls could be calculated. Several assumptions were made for this calculation:

- The textile roll is fully unrolled (100 metres long);
- The beginning or end of the textile roll is fully visible, and
- Static camera position (no (little) camera movement).

Images from the video were extracted to create an overlay for analyses purposes. For the textile roll #1 images were taken which were 11 seconds apart. The shed roof was used as reference. The drop distance was extracted using image pixels. The length of the textile roll (100 metres) was also defined in pixels. The result was a drop speed of 5.2 metres/second. Another textile roll was calculated defined as roll #5.

Calculation of the drop speed was done using six images. This yielded a result of 4.1 metres/second. For calculation a range of drop speeds were taken between 4 and 5.5 metres per second which resulted in a Ballistic Coefficient between 0.252 and 0.363.


Textile roll #5

Textile roll #1

Figure 16: Video image overlay of first and last frame to determine the drop speed of the textile roll. Image transmitted by various media organisations. (Source: unknown)

Wind profile

The wind profile of weather balloon measurements from Rostov on Don Airport was used as input for the trajectory analysis calculations. The last recorded wind on the Flight Data Recorder was 219 degrees at 36 knots.

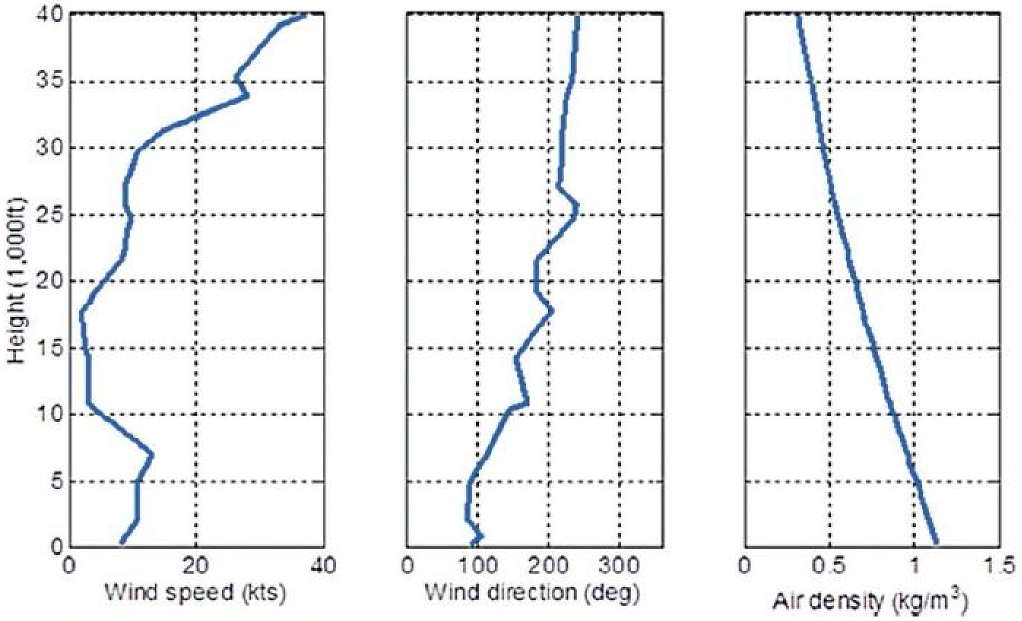


Figure 17: Wind profile used in the ballistic trajectory analysis. (Source: UK Met Office)